Using multiple subexposures for attaining the same S/N as a single exposure

neglecting: dark shot noise, dark fixed pattern noise, fixed pattern noise, sky background noise

R.D. Crisp 4/16/2008

Sk = (S^2 + S * SQRT(S^2 + (S+R^2)*4*K*R^2))/(2*(S + R^2)) expression to compute signal level needed to equal single exposure S/N after K subexposures of Sk level

where:

S = Signal Level after Single Exposure (electrons or DN [be consistent])
K = number of exposures to divide the signal
R = read noise in electrons
Sk =Subexposure signal level required to attain the same S/N as single exposure

example use
you get 7200 electrons in some period of time T

if you were to divide up the exposure into 12 exposures of equal length, how much signal should be in each to get the same S/N?

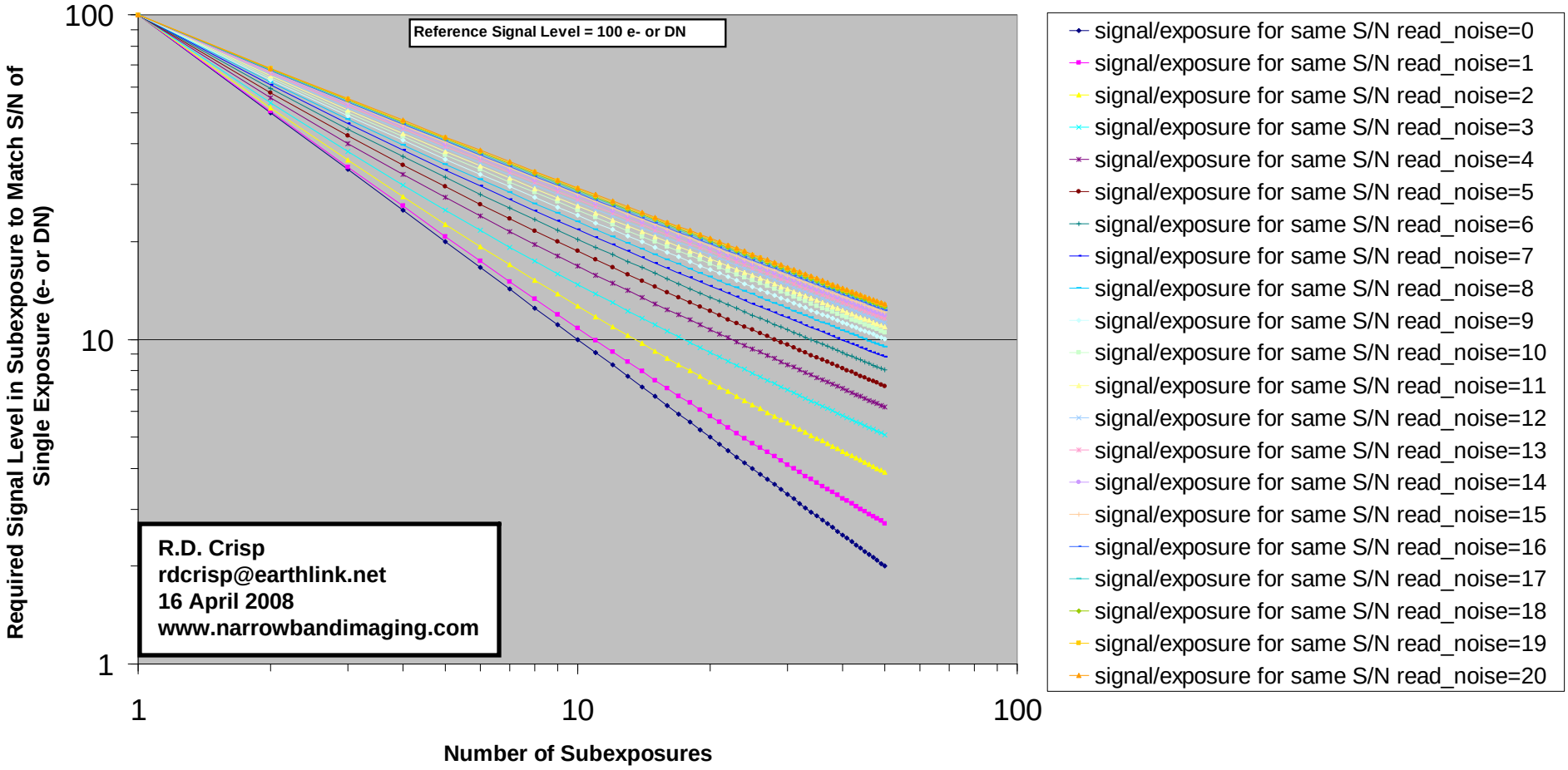
Example solution:

7200	S = Signal Level after Exposure (e- or DN)
12	K = number of exposures to divide the signal
12	R = read noise in electrons or DN
707.89	Sk = Signal level (e- or DN) in subexposures to equal original signal to noise ratio

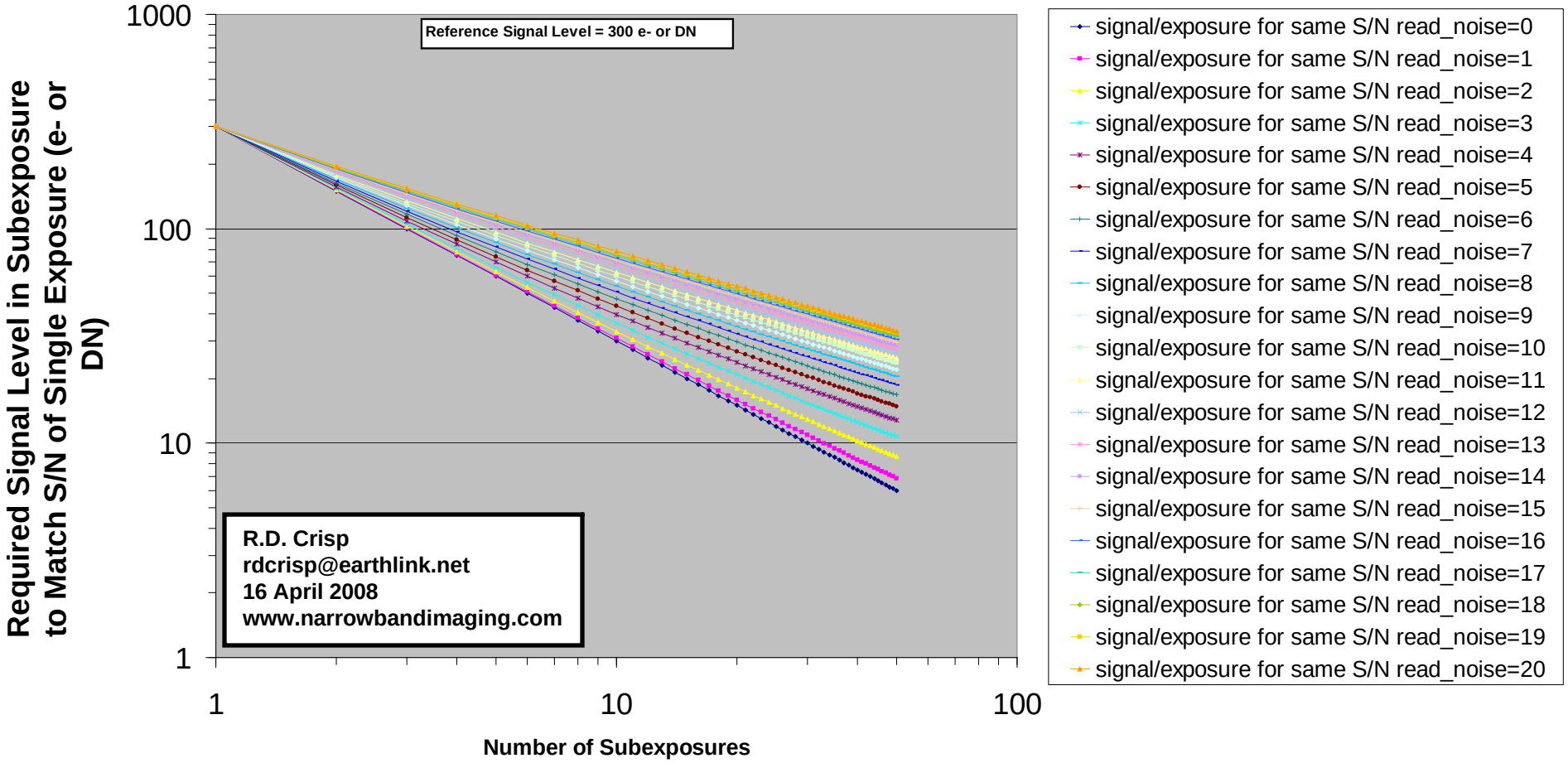
if it takes 10 minutes to collect 600 electrons/pixel, how long should the subexposures be?

10 minutes *	707.89	/	600	=	11.798	minutes per subexposure
				x	12	subexposures
					8494.7	total electrons collected for subexposure case
					7200	total electrons collected for a single exposure
					17.98%	additional signal must be collected for same S/N due to dividing into 12 subexposures with 12 electron read noise

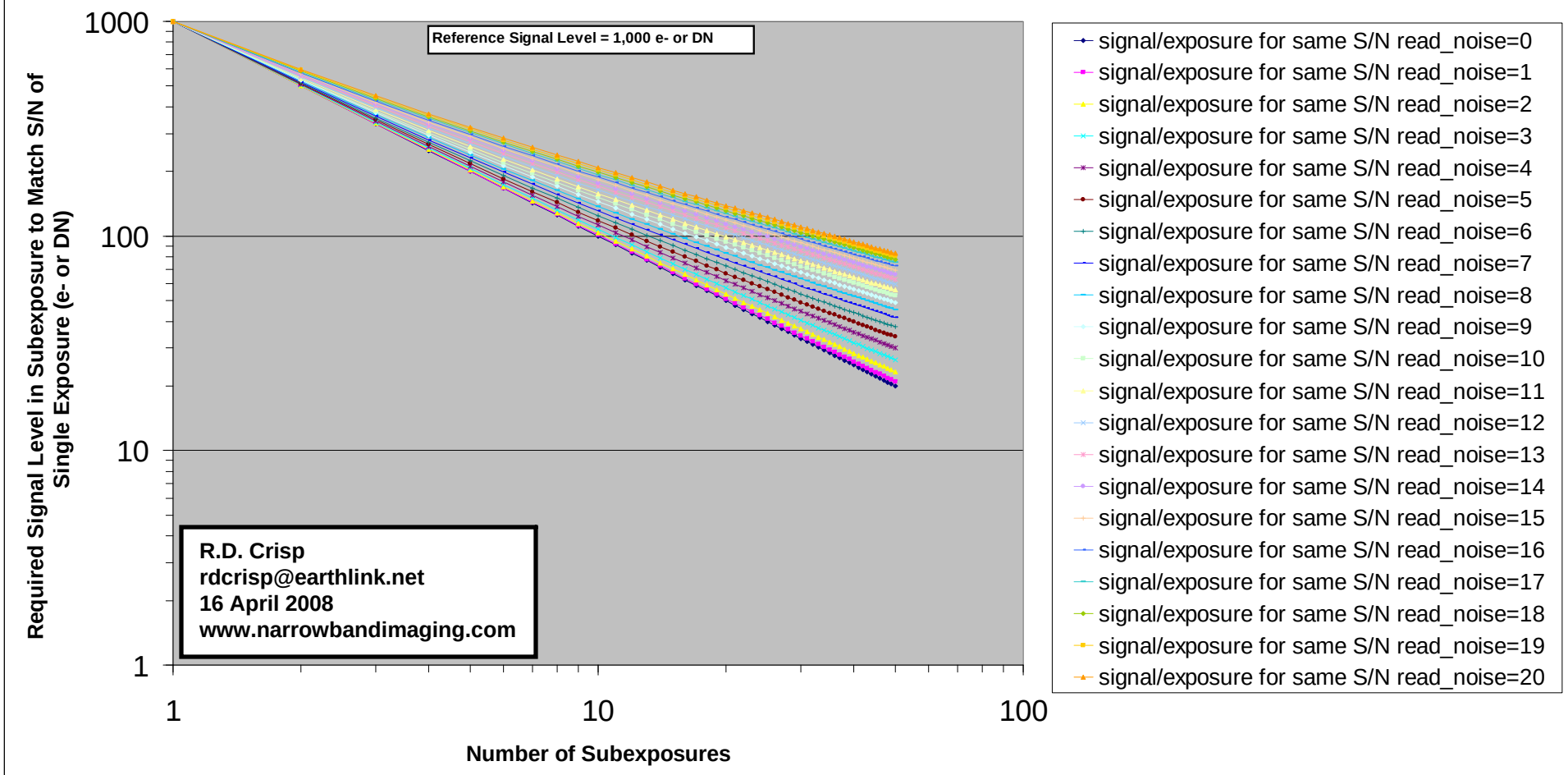
Signal Level in Subexposures Necessary to Match Single Exposure S/N



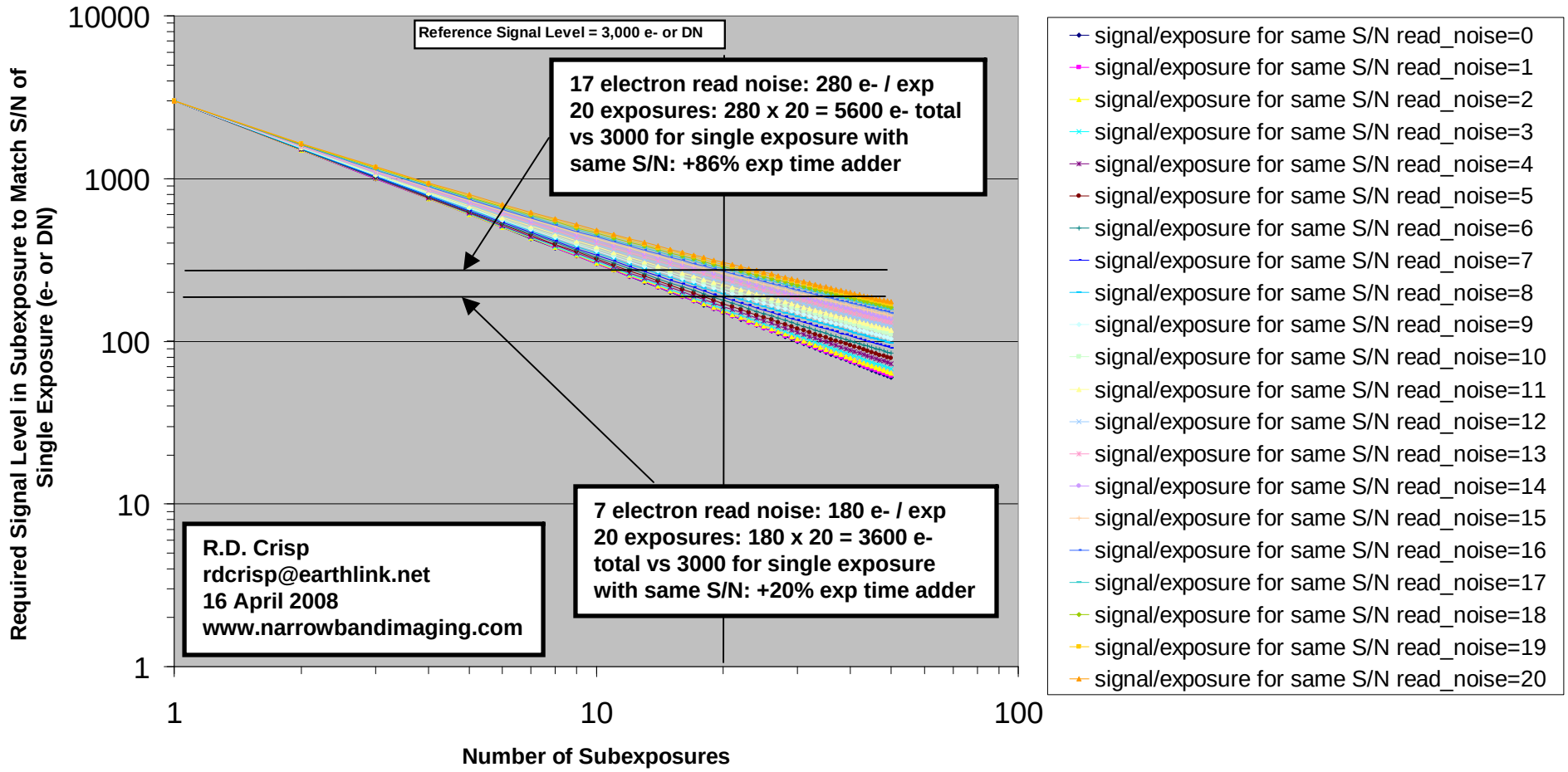
Signal Level in Subexposures Necessary to Match Single Exposure S/N



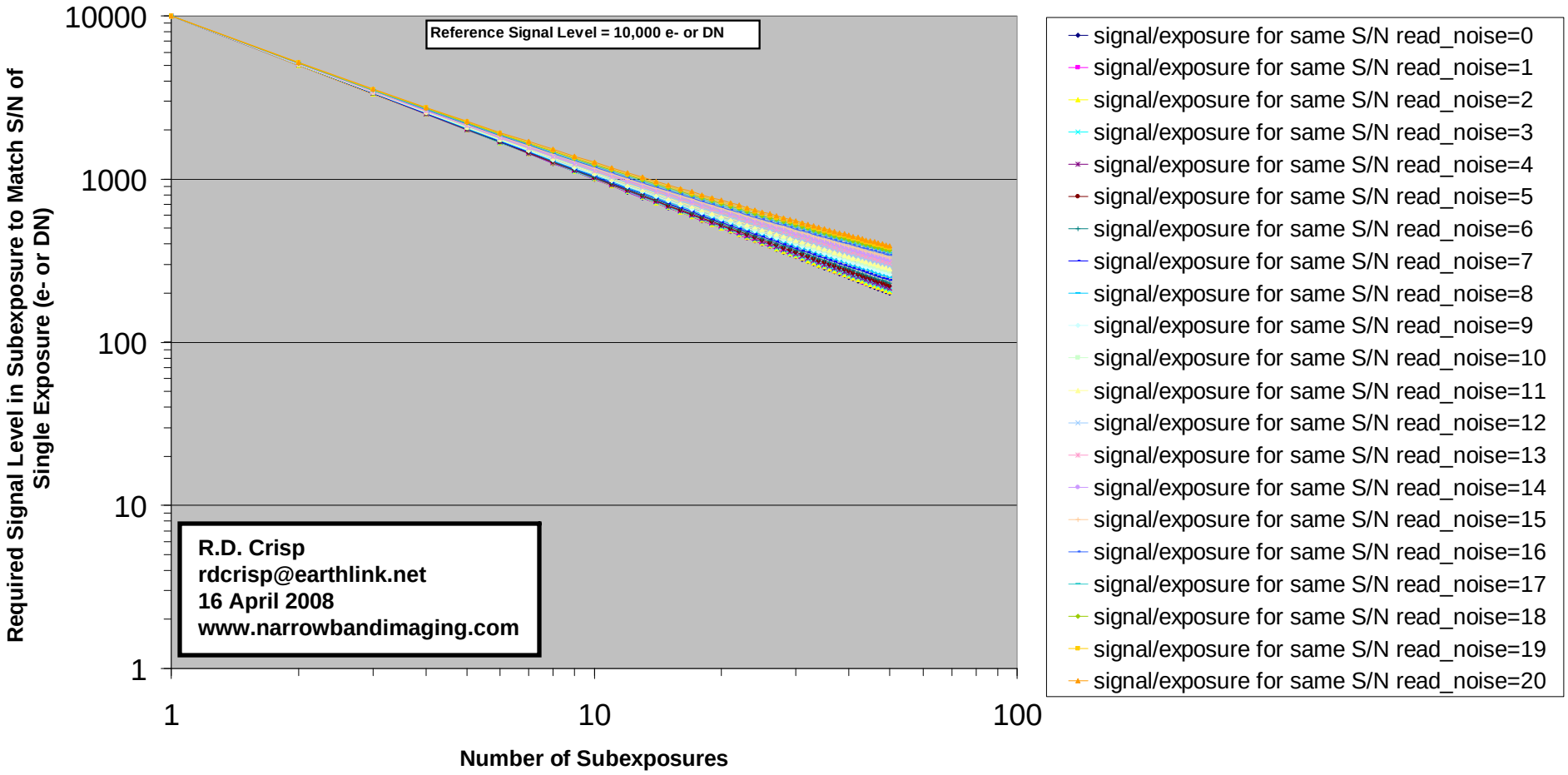
Signal Level in Subexposures Necessary to Match Single Exposure S/N



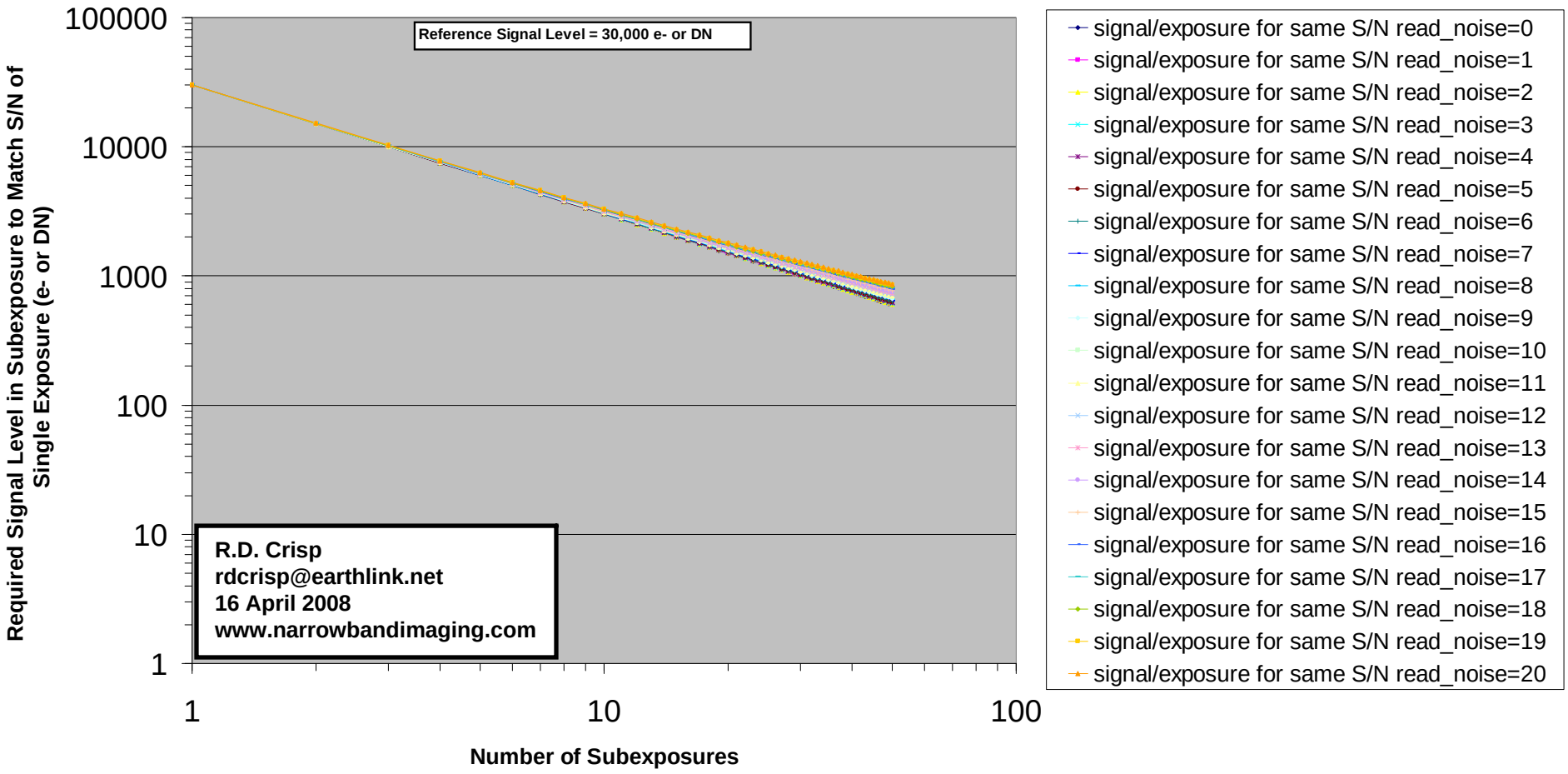
Signal Level in Subexposures Necessary to Match Single Exposure S/N



Signal Level in Subexposures Necessary to Match Single Exposure S/N



Signal Level in Subexposures Necessary to Match Single Exposure S/N



DERIVATION

$$\frac{S}{N} = \frac{\text{Signal}}{\text{Noise}} = \frac{S}{\text{SQRT}(\text{SignalShot}^2 + rd^2)}$$

$$\text{Signal shot} = \sqrt{S}$$

$$\frac{S}{N} = \frac{S}{\sqrt{S + rd^2}}$$

Neglecting:
- Dark shot
- Dark FPN
- FPN

where S = Signal (e^-)
rd = read Noise (e^-)

Break up S into K exposures of EQUAL LENGTH and of value S_K
FIND S_K such that S/N IS SAME AS SINGLE EXPOSURE

$$\frac{S}{N} = \frac{K \cdot S_K}{\sqrt{K(S_K + rd^2)}}$$

SINGLE S/N is the same as single exposure case:

$$\frac{K S_K}{\sqrt{K(S_K + rd^2)}} \equiv \frac{S}{\sqrt{S + rd^2}}$$

SOLVING FOR S_K:

$$S_K = \frac{S^2 \pm S\sqrt{S^2 + 4rd^2 \cdot K(S + rd^2)}}{2K \cdot (S + rd^2)}$$

Craig Sparkock

CHECK the special cases:

SPECIAL Case #1, $r_d = 0$

$$S_K = \frac{s^2 + s\sqrt{s^2 + 0}}{2K(s+0)} = \frac{2s^2}{2Ks} = \frac{s}{K} \checkmark$$

SPECIAL Case #2, $K = 1$

$$S_K = \frac{s^2 + s\sqrt{s^2 + 4r_d^2 \cdot (s + r_d^2)}}{2(s + r_d^2)}$$

$$= \frac{s^2 + \sqrt{s^4 + 4r_d^2 s^2 (s + r_d^2)}}{2(s + r_d^2)}$$

$$= \frac{s^2 + \sqrt{s^4 + 4r_d^2 s^3 + 4r_d^2 s^2}}{2(s + r_d^2)}$$

$$= \frac{s^2 + \sqrt{(s^2 + 2r_d^2 s)^2}}{2(s + r_d^2)}$$

$$= \frac{s^2 + s^2 + 2r_d^2 s}{2(s + r_d^2)} = \frac{2s^2 + 2r_d^2 s}{2(s + r_d^2)}$$

$$= \frac{2s(s + r_d^2)}{2(s + r_d^2)} = s \checkmark$$

CREP 14/MAR/2008

How many exposures of value Signal/K signal level are needed to match the S/N ratio of a single exposure of value Signal?

N = (K^2)*(S/K + R^2)/(S + R^2) expression to compute number of exposures of value S/K to equal the S/N of a single exposure of value S

example use

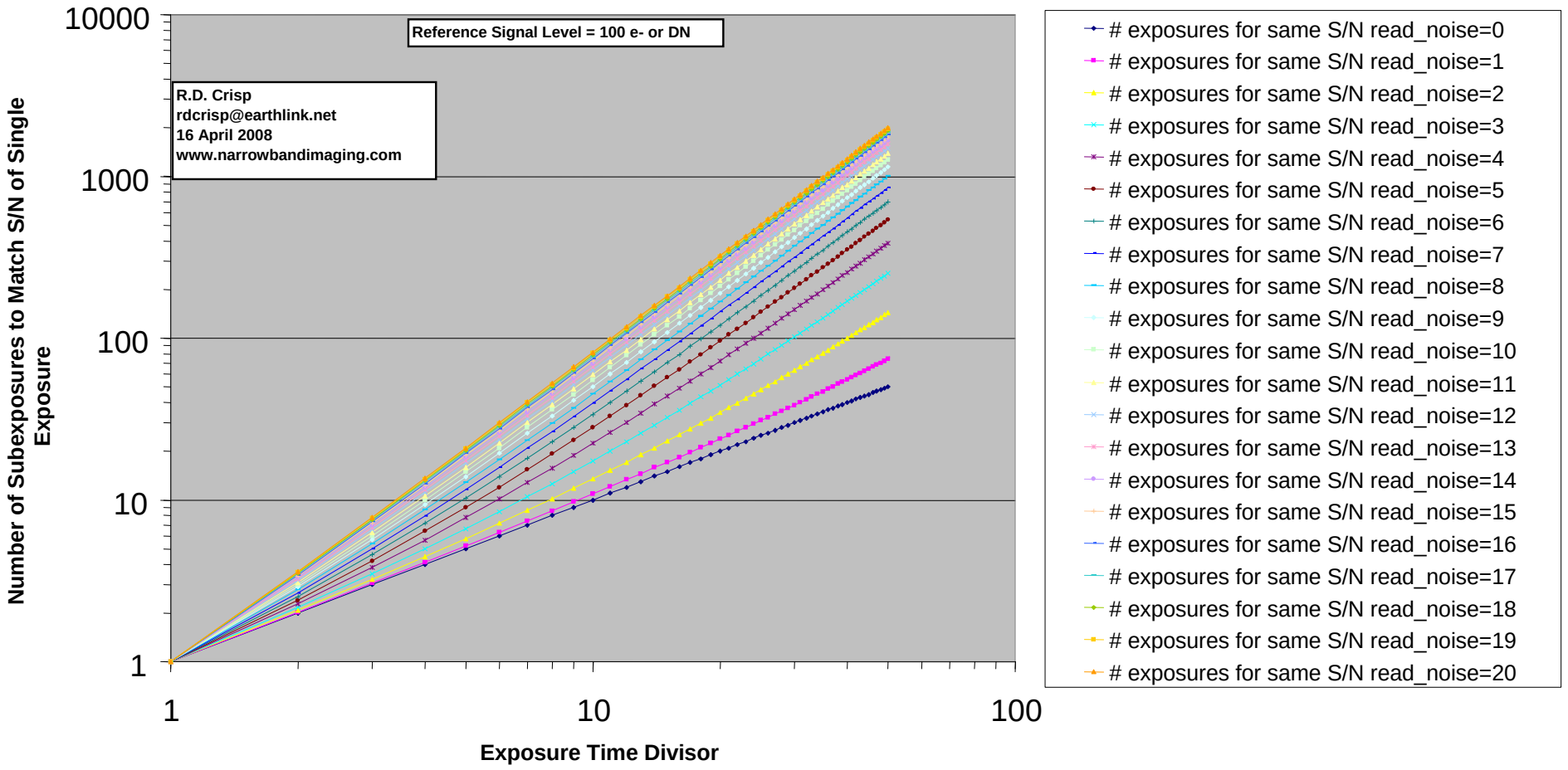
you get 7200 electrons in some period of time T

if you were to divide up the exposure into subexposures of 1/12 the signal of the single exposure, how many such subexposures are needed to attain the same S/N ?

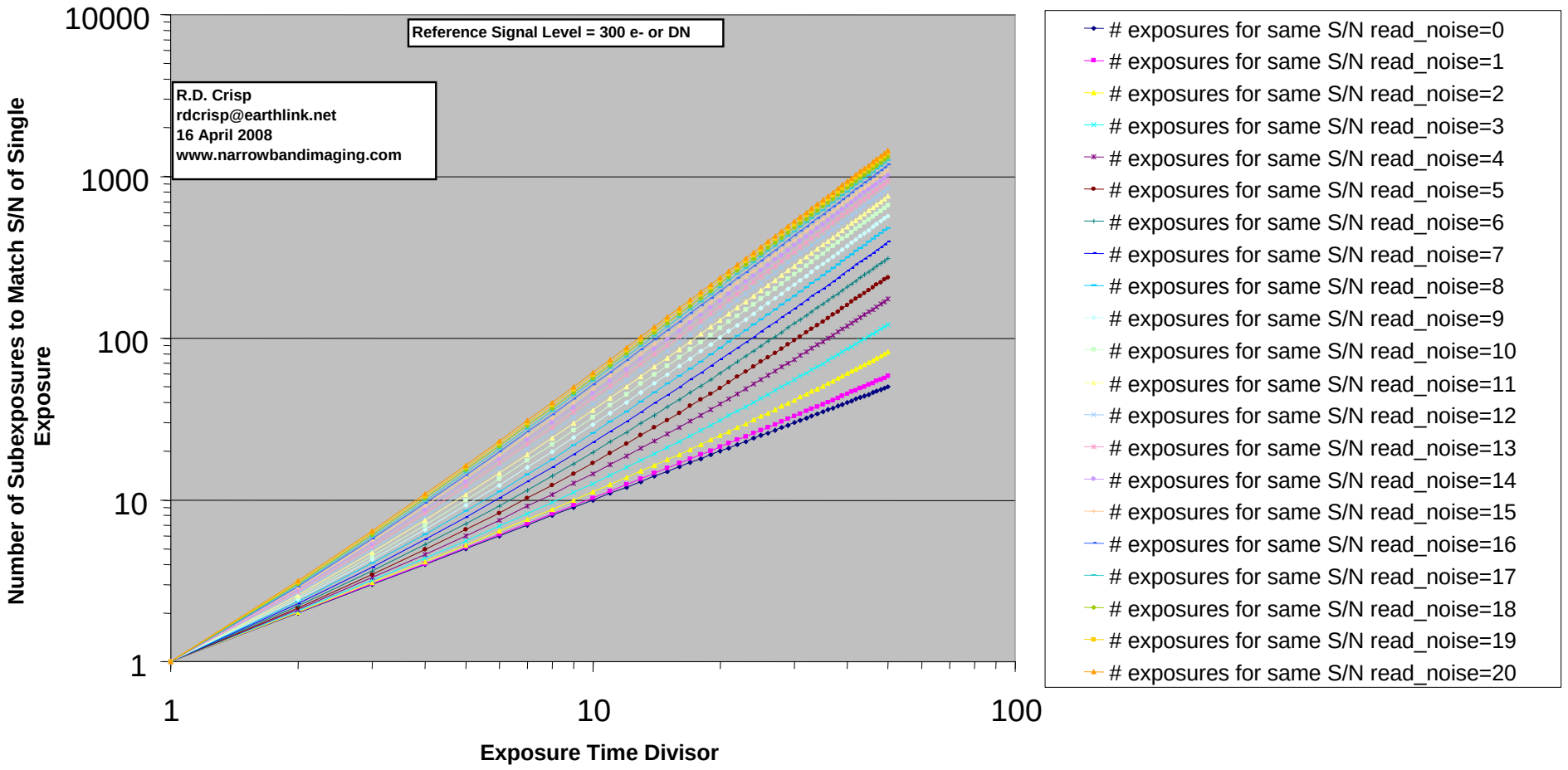
solution:

7200	S = Signal Level after Exposure (electrons)
12	K = number of exposures to divide the signal
12	R = read noise in electrons
14.588	N = Number of subexposures of signal S/K to equal original signal to noise ratio
1.2157	exposure time overhead associated with using N subexposures instead of a single exposure

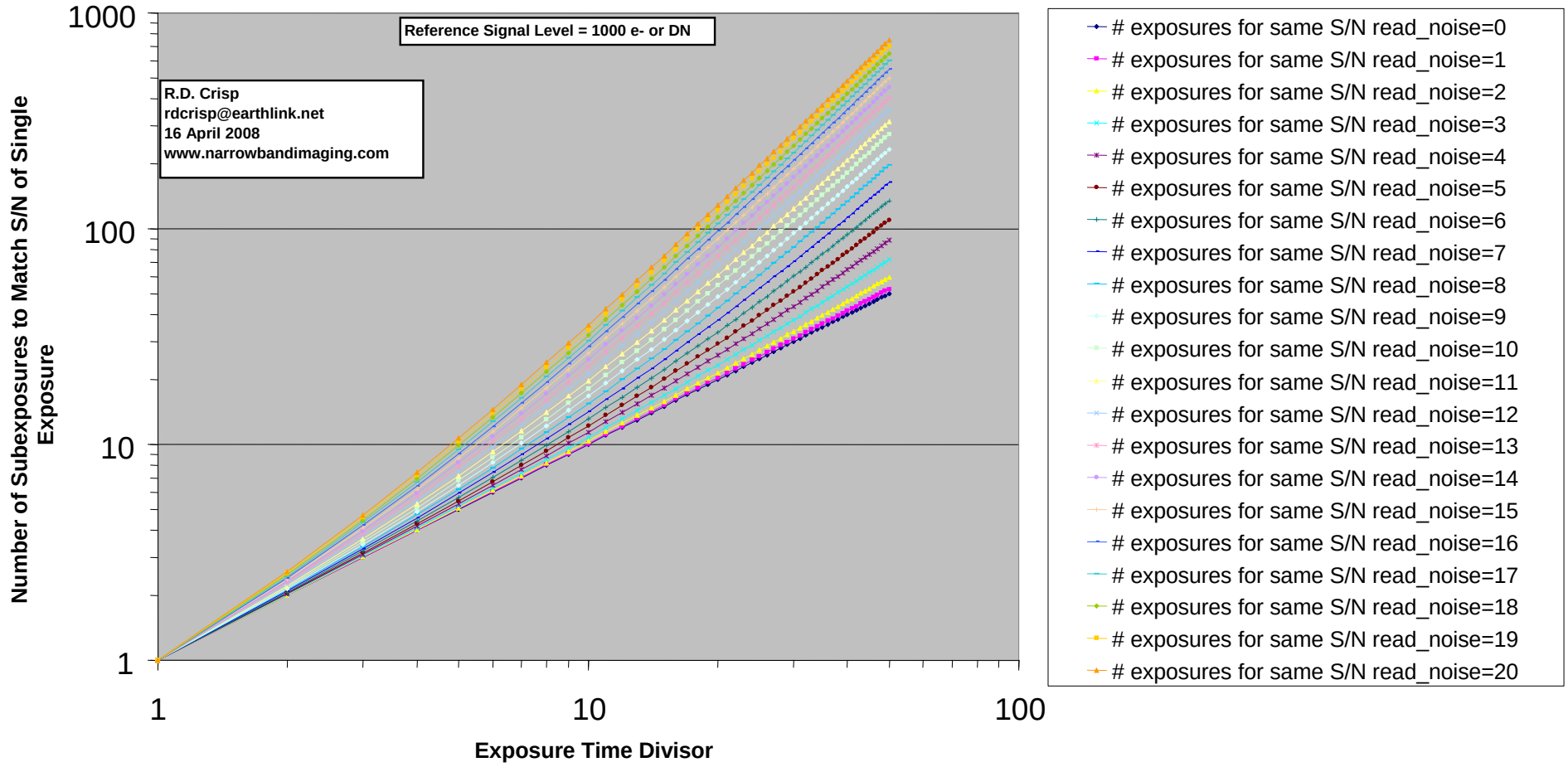
Number of Subexposures Necessary to Match Single Exposure S/N



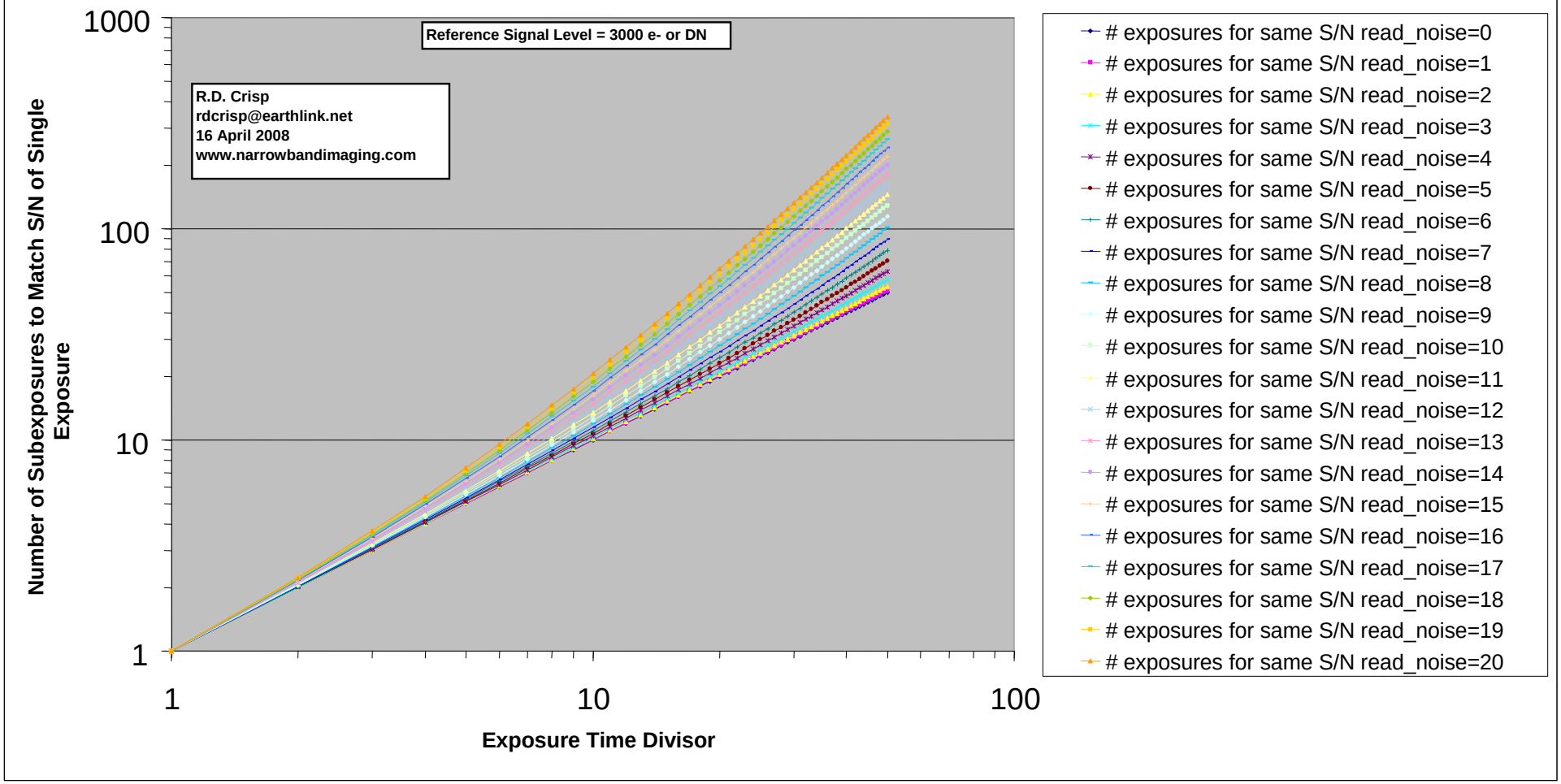
Number of Subexposures Necessary to Match Single Exposure S/N



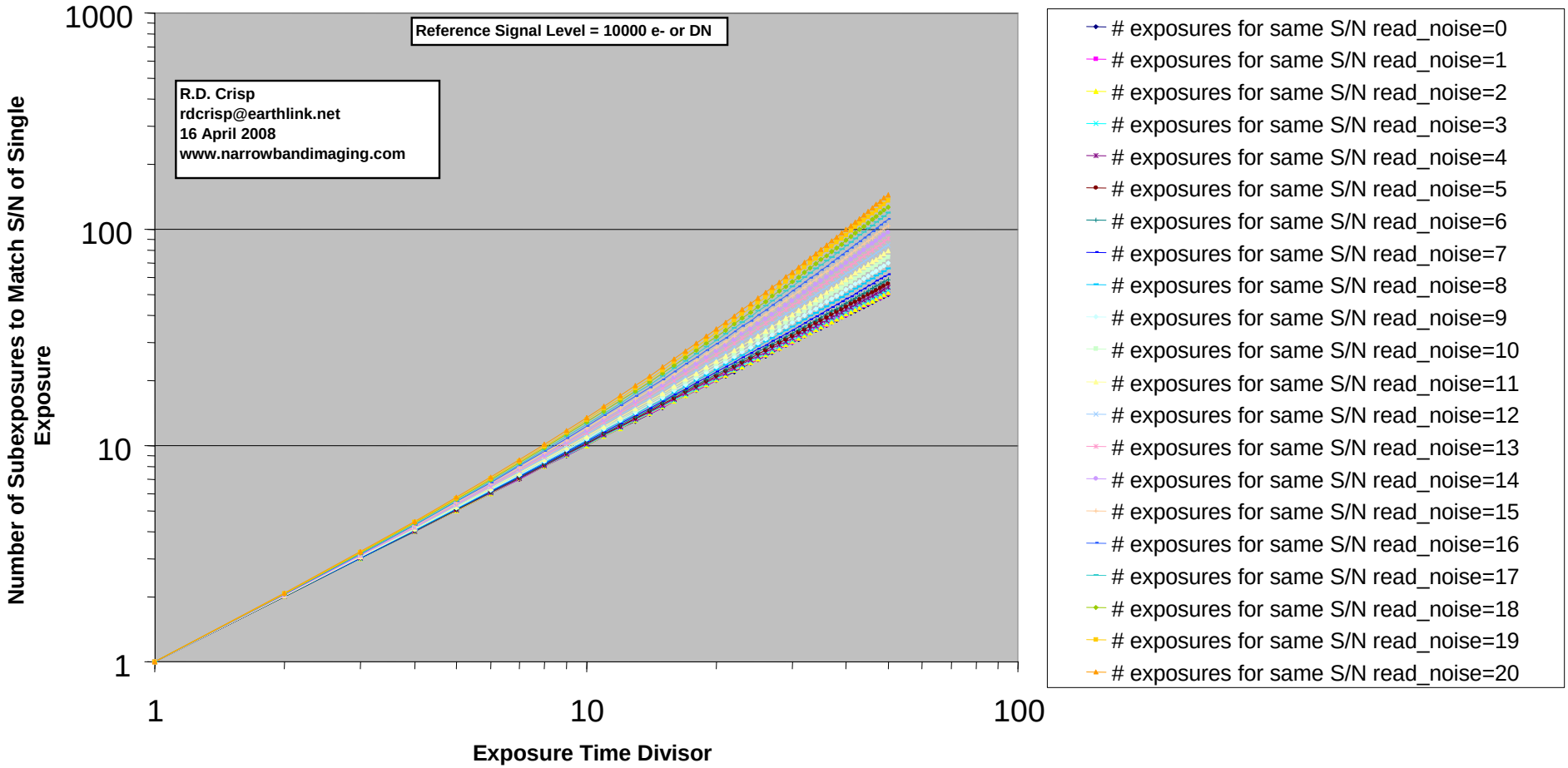
Number of Subexposures Necessary to Match Single Exposure S/N



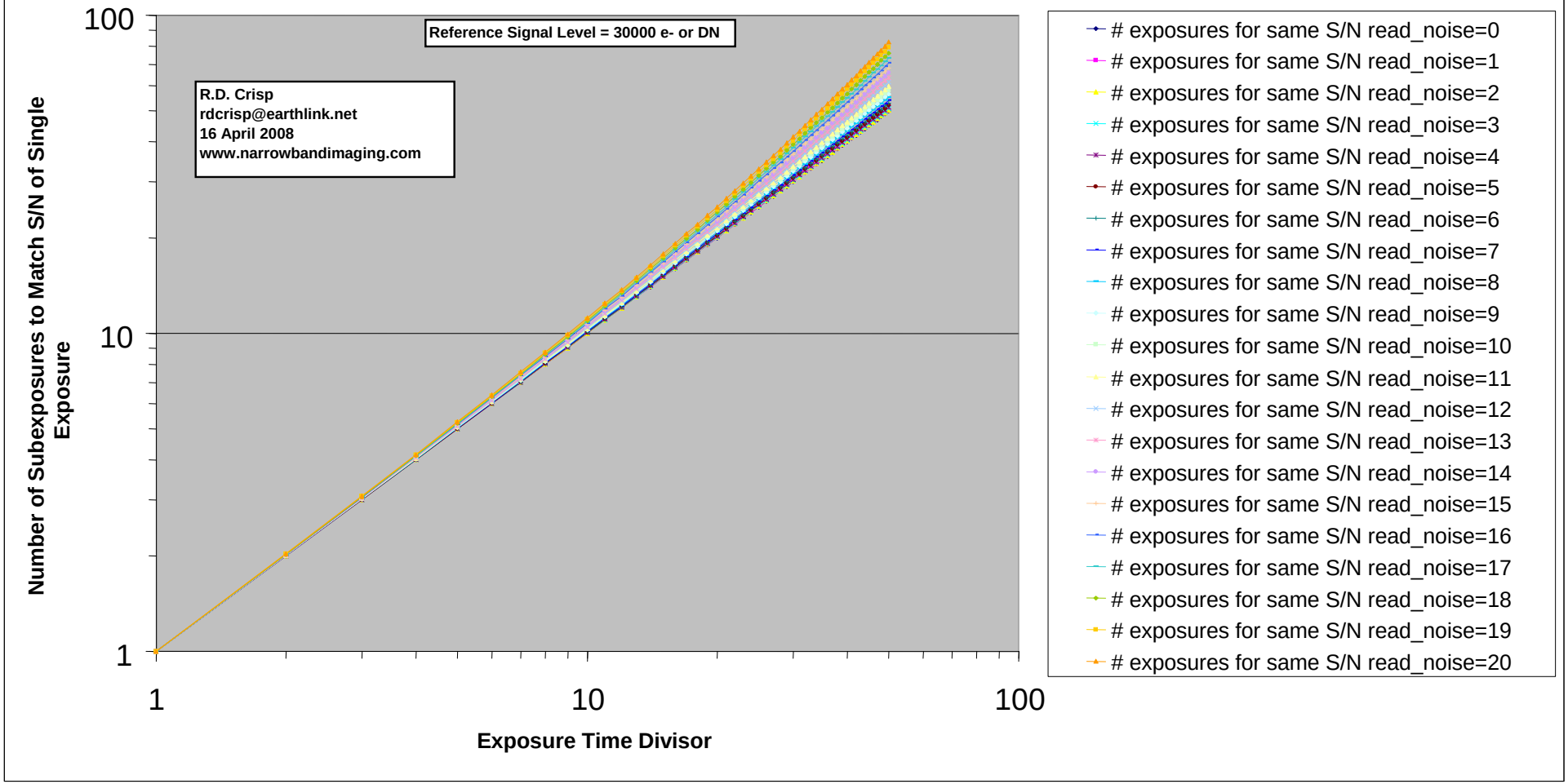
Number of Subexposures Necessary to Match Single Exposure S/N



Number of Subexposures Necessary to Match Single Exposure S/N



Number of Subexposures Necessary to Match Single Exposure S/N



DERIVATION

$$\frac{S}{N} = \frac{S}{\sqrt{\text{SignalShot}^2 + rd^2}}$$

$$\text{signal shot} = \sqrt{S}$$

$$\frac{S}{N} = \frac{S}{\sqrt{S + rd^2}}$$

where S = signal level (e⁻ or DN)
rd = read noise (e⁻ or DN)

Break S into shorter exposures of

$\frac{S}{K}$ value.

$$\frac{S}{N} = \frac{S}{\sqrt{S + K rd^2}}$$ for rd noise = 0, $S/N = \sqrt{S}$

for rd noise > 0, $\frac{S}{N}$ is reduced

from the ideal, $\sqrt{S}$

What number N (exposures), will give an S/N equal to the ideal?

Sub exposure level = $\frac{S}{K}$

N such exposures gives

$$\frac{\frac{N}{K} \cdot S}{\sqrt{\frac{N}{K} S + N rd^2}} = \frac{S}{N} = \frac{S}{\sqrt{S + rd^2}}$$

CKSP
16/MAY/2008

$$\frac{N\,\frac{S}{K}}{\sqrt{N}\cdot\sqrt{\frac{S}{K}+rd^2}} = \frac{\sqrt{N}\cdot\frac{S}{K}}{\sqrt{\frac{S}{K}+rd^2}} = \frac{S}{\sqrt{S+rd^2}}$$

$$\sqrt{N}\,\frac{S}{K}\cdot\sqrt{S+rd^2} = S\cdot\sqrt{\frac{S}{K}+rd^2}$$

$$\sqrt{N} = \frac{K\,S\,\sqrt{\frac{S}{K}+rd^2}}{S\,\sqrt{S+rd^2}} = K\sqrt{\frac{\frac{S}{K}+rd^2}{S+rd^2}}$$

$$\boxed{N = K^2\cdot\left(\frac{\frac{S}{K}+rd^2}{S+rd^2}\right)} \text{ derived expression}$$

CHECK SPECIAL CASES

$rd = 0$

$$N = K^2\cdot\left(\frac{\frac{S}{K}+0}{S+0}\right) = K^2\,\frac{S}{K}\,\frac{1}{S} = K \quad \checkmark$$

$K = 1$

$$N = 1\cdot\frac{\frac{S}{1}+rd^2}{S+rd^2} = 1 \quad \checkmark$$

CRSP
4/16/2002