

# Quantizing Noise in a Digital Imaging System

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Quantizing noise arises from the discrete nature of the sampling/conversion process of the A/D Converter (ADC). If a continuous linear ramp is applied to the input of the ADC you get discrete approximations of the input values from the output of the converter. There is an error associated with the output value being only able to assume discrete instead of continuous values. This error is separate and distinct from other sources of error in an ADC.

It is important to note that the RMS value of the Quantizing noise is fixed in terms of DN. The magnitude of the Quantizing noise is computed by integrating the square of the value of the error (actual ADC discrete response minus the continuous response of an ideal ADC) over the interval of one DN. When solved the value is derived to be:

$$QN(DN) = 1/\sqrt{12} \quad (\text{RMS DN})$$

An interesting property of Quantizing noise is that it is a fixed value when measured in DN for any ADC of any finite resolution. It is always equal to  $1/\sqrt{12}$ , which is about 2/7 of a DN. Because the value is fixed in terms of DN, the value in electrons will vary depending on the camera gain. For any particular camera gain, K (electrons/DN) the RMS value of the quantizing noise is constant and is equal to:

$$QN(e^-) = K/\sqrt{12} \quad (\text{RMS electrons})$$

Depending on the camera gain relative to the read noise, the quantizing noise can be prominently seen in an image such as in Figure 1 below.

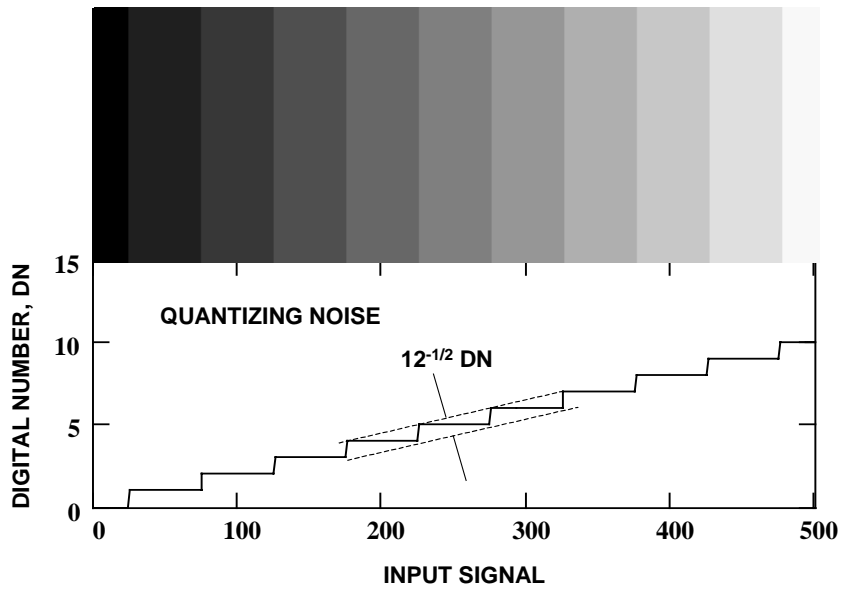
Like the noise from other uncorrelated sources, the Quantizing noise, in electrons, is added in quadrature with the noise from other sources to compute the total system noise.

Therefore for a camera with a read noise  $R(e^-)$ , an image shot noise  $N = \sqrt{\text{Signal}(e^-)}$  and Quantizing noise  $QN(e^-) = K/\sqrt{12}$ , you get (ignoring other noise sources for now):

$$\text{noise (RMS, } e^-) = \sqrt{R(e^-)^2 + (\sqrt{\text{signal}(e^-)})^2 + (K/\sqrt{12})^2} = \sqrt{R(e^-)^2 + \text{signal}(e^-) + (K^2/12)}$$

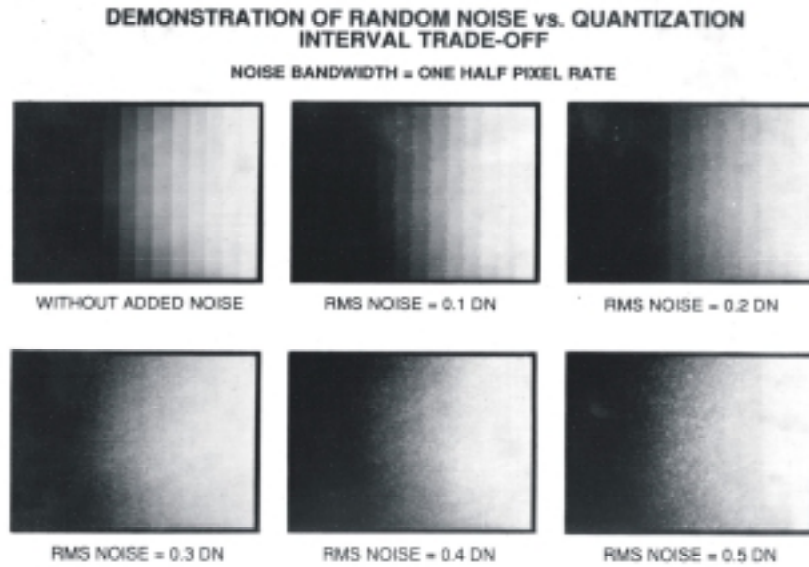
As a camera designer you have control of the camera gain and can therefore set as a design parameter the proportion of the total noise of the camera that will be due to Quantizing noise.

### ADC QUANTIZING NOISE



Source: Janesick

Figure 1.



Source: Janesick

Figure 2.

Figure 2 above shows that the quantizing noise of the camera becomes insignificant when the numerical value of the Read noise in units of DN is greater or equal to about  $0.5 * \text{Quantizing noise in DN}$ . According to Janesick, when the read noise in electrons is approximately equal to the Quantizing noise in electrons, the Quantizing noise no longer is a factor.

Example:

A camera has a read noise of 5 electrons and a full well of 100,000 electrons.

- A) Compute the optimal value of camera gain to avoid Quantizing noise in the resulting image and
- B) determine the dynamic range of the camera, and
- C) how many bits of resolution the A/D converter requires to encode the full well.
- D) Next, compute the read noise in DN and compare with the Quantizing noise in DN.
- E) Finally compute the noise for a signal level of 50 electrons with and without the quantizing noise contribution
- F) and compare for camera gain of 5 and camera gain of 20 electrons/DN.

Solution:

- A) Set the camera gain to equal the numerical value of the read noise,  $5e^-$ , so  $K=5e^-/DN$ .
- B) The dynamic range is  
 $(\text{full well})/(\text{read noise}) = 100000 e^- / 5 e^- = 20,000:1$  or  
 $20 * \text{Log}_{10}(20,000) = 86.02\text{dB}$   
 $N_{adc} = 20,000$  states, so  $\text{ADCbits} = \log(20000)/\log(2) = 14.28$  bits  
 so you need a 15 bit ADC.
- D) Read noise( $e^-$ ) =  $5e^-$   
 Read noise (DN) =  $\text{Read noise}(e^-)/K = 5e^-/5e^-/DN = 1DN$   
 Quantizing noise (DN) =  $1/\sqrt{12} = 0.2886$  or about 2/7ths of a DN
- E)  $K=5\text{electrons}/DN$  case:  
 $\text{Noise}(e^-) = \sqrt{R(e^-)^2 + \text{Signal}(e^-) + K^2/12}$  (electrons)  
 $\text{Noise}(e^-) = \sqrt{5^2 + 50 + 5^2/12}$  electrons  
 $\text{Noise}(e^-) = \sqrt{25 + 50 + 25/12} = \sqrt{77.5} = 8.803$  electrons  
 (counting the Quantization noise)  
  
 Or without the Quantization noise:  
 $\text{Noise}(e^-) = \sqrt{75} = 8.661$  electrons
- F)  $K=20\text{electrons}/DN$  case:  
 $\text{Noise}(e^-) = \sqrt{R(e^-)^2 + \text{Signal}(e^-) + K^2/12}$  (electrons)

Noise(e-) =  $\sqrt{5^2 + 50 + 20^2/12}$  electrons  
Noise(e-) =  $\sqrt{25 + 50 + 400/12} = \sqrt{108.33} = 10.408$  electrons  
(counting the Quantization noise)

Or without the Quantization noise:  
Noise(e-) =  $\sqrt{75} = 8.661$  electrons